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FIRE CONTROL METHODOLOGY FOR AN ACCELERATING TARGET

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Methodology in this report will be used in future parametric studies.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The effect of sensor errors in the line-of-sight rate is determined by relating these errors through a data smoothing and prediction scheme to errors in the transverse velocity and acceleration. The objective is to derive the target miss distance as a function of important design and tactical variables (i.e., data rate, smoothing time, firing delay, velocity and acceleration).

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INTRODUCTION

The purpose of the report was to determine the effect of sensor errors in the line-of-sight rate on the miss distance for an accelerating target. Since ARRADCOM is responsible for developing fire control systems for antitank application, it was decided to develop a methodology which would accept the basic error per measurement; and then by using a number of these measurements in succession to develop a smoothing and prediction scheme which would quantify important parameters at the time of launch. These estimated parameters at the time of fire are important in determining the gun lead angle and the resulting miss distance at the target. Also, measurements taken by the sensors are not perfect and result in errors in the gun lead angle and therefore in miss distance.

BACKGROUND

Previous studies have concentrated on various filter models and classes of moving vehicle using actual target dynamic data. Continuous tracking was assumed with no parametric sensitivity analysis to show impact errors as a function of short tracking times and sensor noise (ref 1-3). This analysis presents work done from January through March of 1979.

METHODOLOGY

The design methodology is shown in figure 1.

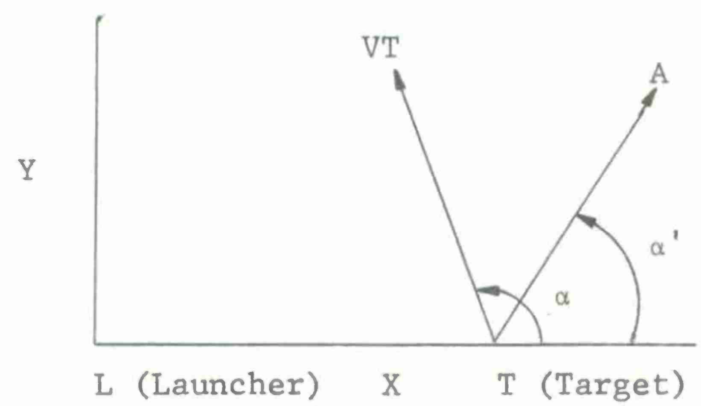

Figure 1. Target and launcher geometry at time of fire.

Since $\dot{\theta}$ is the Y component of the velocity of the target divided by range X,

$$\dot{\theta} = VT \cdot SIN\ \alpha\ /X \tag{1}$$

It is assumed that the target is following a path in the prelaunch phase of $\ddot{\theta}$ which equals a constant prior to firing. As a result, $\dot{\theta}$ will vary linearly with time. A least squares fit of the data is therefore a best approximation by drawing a straight line through the measured data points in the prelaunch phase. Employing linear regression techniques, one obtains (see appendix A)

$$E\dot{\theta} = \frac{1}{Det} \frac{\sum_{n=1}^{NMAX} C_n \cdot \dot{\theta}N_n}{\dot{\theta}} \tag{2}$$

Where C_n is defined by (appendix A)

$$C_n = \left(\sum_{n=1}^{NMAX} t_n^2\right) - t_n \cdot \left(\sum_{n=1}^{NMAX} t_n\right) + \left(t_n \cdot NMAX - \sum_{n=1}^{NMAX} t_n\right) \cdot TF \tag{3}$$

and where the summation signs are from n=1 to n=NMAX. Therefore,

$$Det = NMAX \cdot \sum_{n=1}^{NMAX} t_n^2 - \left(\sum_{n=1}^{NMAX} t_n\right)^2 \tag{4}$$

One then obtains solutions for the next two equations (see appendix A).

The first equation states that at intercept, the X distance that the projectile travels away from the launcher, TINT · VP · COS AP, must equal the distance of the target form the launcher. This distance is the initial distance at time of fire, X_o, plus the distance travelled due to the initial velocity in the X direction TINT · VT · COS α, and the distance travelled due to the component of target acceleration, A COS α' $\frac{1}{2}$ $TINT^2$ and results in

$$\mathrm{TINT} \cdot \mathrm{VP} \cdot \mathrm{COS\ AP} = X_o + \mathrm{VT} \cdot \mathrm{COS}\ \alpha \cdot \mathrm{TINT} + \tfrac{1}{2}(\mathrm{A\ COS}\ \alpha') \cdot \mathrm{TINT}^2 \qquad (5)$$

Since at intercept the distance travelled by the projectile in the y direction must equal the distance travelled by the target in the y direction, one obtains

$$\mathrm{TINT} \cdot \mathrm{VP} \cdot \mathrm{SIN\ AP} = \mathrm{VT} \cdot \mathrm{SIN}\ \alpha \cdot \mathrm{TINT} + \tfrac{1}{2}\mathrm{A} \cdot \mathrm{SIN}\ \alpha' \cdot \mathrm{TINT}^2 \qquad (6)$$

Equations 5 and 6 are then solved for TINT and AP.

One uses the nominal intercept time, TINT, to calculate the miss distance. These subsequent calculations are based upon the launcher measurements and measurement errors. The line-of-sight rate error is the main focus of interest. Therefore, errors arising from parameters, other than the line of sight rate error, are not considered here. The influence of the error on transverse velocity and acceleration estimates are, however, assessed, since poor estimates will result in an increased miss distance at the target. For example,

$$\mathrm{MD} = \mathrm{TINT} \cdot (\mathrm{VTJ} - \mathrm{VTJP}) + \frac{(\mathrm{TINT})^2}{2} \cdot (\mathrm{AJ} - \mathrm{AJP}) \qquad (7)$$

By definition of $E\dot{\theta}$ (appendix A)

$$E\dot{\theta} = (\dot{\theta}P - \dot{\theta})/\dot{\theta} \qquad (8)$$

and

$$E\ddot{\theta} = (\ddot{\theta}P - \ddot{\theta})/\ddot{\theta} \qquad (9)$$

since

$$\dot{\theta} = \mathrm{VTJ}/X_o \qquad (10)$$

and

$$\dot{\theta}P = \mathrm{VTJP}/X_o \qquad (11)$$

substituting (10) and (11) into (8)

$$E\dot{\theta} = \left(\frac{VTJP}{X_o} - \frac{VTJ}{X_o}\right) / \dot{\theta} \tag{12}$$

or

$$E\dot{\theta} = (VTJP - VTJ)/(X_o \cdot \dot{\theta}) \tag{13}$$

or

$$VTJP - VTJ = E\dot{\theta} \cdot X_o \cdot \dot{\theta} \tag{14}$$

From Greenwood, page 37, equation 2-23 (ref. 4), $e_r = i$ and $e_\phi = j$. Solving the j component for $\ddot{\phi}$, i.e., $\ddot{\theta}$ in the nomenclature of this report

$$\ddot{\theta} = (AJ - 2 \cdot \dot{\theta} \cdot VTI)/X_o \tag{15}$$

and similarly

$$\ddot{\theta}P = (AJP - 2\dot{\theta}P \cdot VTI)/X_o \tag{16}$$

Substituting (15) and (16) in (9) yields

$$E\ddot{\theta} = \frac{(AJP - 2\dot{\theta}P \cdot VTI) - (AJ - 2\dot{\theta} \cdot VTI)}{X_o\ddot{\theta}} \tag{17}$$

or

$$E\ddot{\theta} = \frac{(AJP - AJ) + 2VTI\ (\dot{\theta} - \dot{\theta}P)}{\left(X_o \cdot \ddot{\theta}\right)} \tag{18}$$

Substituting (8) into (18), yields

$$E\ddot{\theta} = \left[(AJP - AJ) + 2 \cdot VTI(-E\dot{\theta} \cdot \dot{\theta})\right] / (X_o \cdot \ddot{\theta}) \tag{19}$$

From (19) and solving for AJ - AJP,

$$AJ - AJP = -E\ddot{\theta} \cdot X_o \cdot \ddot{\theta} + 2 \cdot VTI(-E\dot{\theta} \cdot \dot{\theta}) \tag{20}$$

From (13),

$$VTJ - VTJP = -E\dot{\theta} \cdot X_o \cdot \dot{\theta} \tag{21}$$

Substituting (20) and (21) into (7), one finds

$$MD = TINT \cdot (-E\dot{\theta} \cdot X_o \cdot \dot{\theta}) + \frac{TINT^2}{2} \left[(-E\ddot{\theta} \cdot X_o\, \ddot{\theta}) + 2 \cdot VTI(-E\dot{\theta} \cdot \dot{\theta})\right] \tag{22}$$

or

$$MD = E\dot{\theta} \cdot \left[-TINT \cdot X_o\, \dot{\theta} - \frac{TINT^2}{2} \cdot 2 \cdot VTI \cdot \dot{\theta}\right] + E\ddot{\theta}\left[-\frac{TINT^2}{2} \cdot X_o\, \ddot{\theta}\right] \tag{23}$$

or

$$MD = E\dot{\theta}\left[(TINT \cdot X_o + \frac{TINT^2}{2} \cdot 2 \cdot VTI)\ (-1)\dot{\theta}\right] + E\ddot{\theta}\left[-\frac{TINT^2}{2} \cdot X_o\, \ddot{\theta}\right] \tag{24}$$

From (24) let

$$K_1 = \left[(TINT \cdot X_o + \frac{TINT^2}{2} \cdot 2 \cdot VTI)\ (-1)\ \dot{\theta}\right] \tag{25}$$

$$K_2 = \left[-\frac{TINT^2}{2} \cdot X_o\, \ddot{\theta}\right] \tag{26}$$

Inserting (25) and (26) into (24) yields

$$MD = E\dot{\theta} \cdot K_1 + E\ddot{\theta} \cdot K_2 \tag{27}$$

Using linear regression techniques (see appendix B)

$$E\ddot{\theta} = \frac{1}{\ddot{\theta}}\left\{\frac{1}{Det} \sum_{n=1}^{NMAX} \dot{\theta}_{N_n} \left[t_n \cdot NMAX - \sum_{n=1}^{NMAX} t_n\right]\right\} \tag{28}$$

Let

$$CTDD_n = \left[t_n \cdot NMAX - \sum_{n=1}^{NMAX} t_n\right] \tag{29}$$

Inserting (29) into (28) yields

$$E\ddot{\theta} = \frac{1}{\ddot{\theta}} \cdot \frac{1}{Det} \sum_{n=1}^{NMAX} \dot{\theta}_{N_n} \cdot CTDD_n \qquad (30)$$

(2) and (30) in (27)

$$MD = \frac{K_1}{Det} \frac{\sum_{n=1}^{NMAX} C_n \cdot \dot{\theta}_{N_n}}{\ddot{\theta}}$$

$$+ \frac{K_2}{\ddot{\theta} \cdot Det} \sum_{n=1}^{NMAX} \dot{\theta}_{N_n} \cdot CTDD_n \qquad (31)$$

or

$$MD = \sum_{n=1}^{NMAX} \dot{\theta}_{N_n} \left(\frac{Cn \cdot K_1}{Det \cdot \ddot{\theta}} + \frac{CTDD_n \cdot K_2}{\ddot{\theta}\ Det} \right) \qquad (32)$$

$$(SIGMA\quad MD)^2 = \sum_{n=1}^{NMAX} \sigma^2\ \dot{\theta}_{N_n} \left(\frac{C_n \cdot K_1}{Det \cdot \ddot{\theta}} + \frac{CTDD_n \cdot K_2}{\ddot{\theta} \cdot Det} \right)^2 \qquad (33)$$

$$SIGMA\quad MD = \left[\sum_{n=1}^{NMAX} \sigma^2\ \dot{\theta}_{N_n} \left(\frac{C_n \cdot K_1}{Det \cdot \ddot{\theta}} + \frac{CTDD_n K_2}{\ddot{\theta} \cdot Det} \right)^2 \right]^{\frac{1}{2}} \qquad (34)$$

If $\sigma^2 \dot{\theta}_{N_n}$ is constant with n, by substituting $\sigma^2 \dot{\theta}_N$, one obtains

$$SIGMA\quad MD = \sigma\ \dot{\theta}_N \left[\sum_{n=1}^{NMAX} \left(\frac{C_n \cdot K_1}{Det \cdot \ddot{\theta}} + \frac{CTDD_n \cdot K_2}{\ddot{\theta} \cdot Det} \right)^2 \right]^{\frac{1}{2}} \qquad (35)$$

CONCLUSIONS

By using this approach, it has been possible to present the main statistical concepts and to relate the variance in the tracking rate noise to the variance in the mil error at the target. This concept takes into account the smoothing, prediction, projectile flight, and a second order prediction scheme. Effects of other important variables such as data rate, smoothing time, firing delay (due to remaining computations and final aiming corrections required after the last measurement), target velocity, target velocity direction, target acceleration, target acceleration direction, projectile velocity, range and gun lead angle have also been included.

RECOMMENDATIONS

Analyses should be performed which show the effects of the above parameters. Further recommendations will be made pending completion of these parametric investigations.

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4. Donald T. Greenwood, Prentice-Hall Inc., <u>Principles of Dynamics</u>, p 37.

APPENDIX A. DERIVATION OF ERROR IN LINE-OF-SIGHT RATE

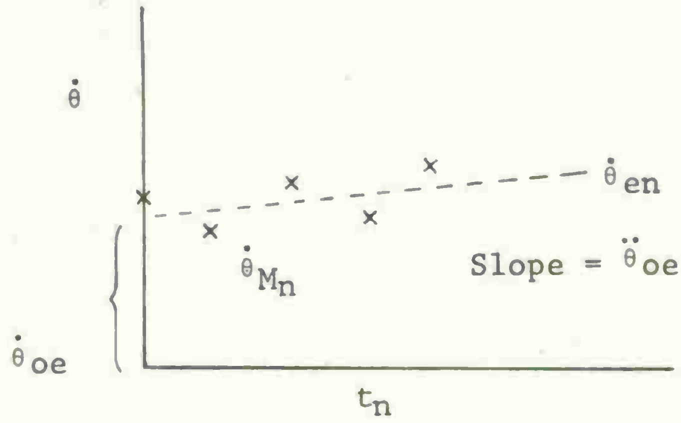

Figure A-1. Smoothing process.

$$\dot{\theta}_{M_n} = \dot{\theta}_o + \ddot{\theta}_o \cdot t_n + \dot{\theta}_{N_n} \qquad \text{(A-1)}$$

$$\dot{\theta}_{en} = \dot{\theta}_{o_e} + \ddot{\theta}_{o_e} \cdot t_n \qquad \text{(A-2)}$$

(where $t_n = Dt \cdot (n-1)$)

M = measured value
e = Estimated value
N = noise error'
n = n-th measurement

For best fit of dash line to the $\dot{\theta}_{M_n}$ points, the sum of the differences, squared

$$\equiv S = \sum_{n=1}^{nx} (\dot{\theta}_{M_n} - \dot{\theta}_{en})^2 \qquad \text{(A-3)}$$

By substituting (A-1) and (A-2) into (A-3), one has

$$S = \sum_{n=1}^{nx} \left[(\dot{\theta}_{oe} - \dot{\theta}_{o}) + (\ddot{\theta}_{oe} - \ddot{\theta}_{o}) t_n - \dot{\theta}_{N_n} \right]^2 \qquad \text{(A-4)}$$

To minimize S with respect to the two variables, $\dot{\theta}_{oe}$ and $\dot{\theta}_{oe}$, take

$$\frac{\partial s}{\partial \dot{\theta}_{oe}} = 0 = \sum_{n=1}^{nx} 2 \left[(\dot{\theta}_{oe} - \dot{\theta}_{o}) \right.$$

$$\left. + (\ddot{\theta}_{oe} - \ddot{\theta}_{o}) t_n - \dot{\theta}_{N_n} \right]^{1} \cdot 1 \qquad \text{(A-5)}$$

$$\frac{\partial S}{\partial \ddot{\theta}_{oe}} = 0 = \sum_{n=1}^{nx} 2 \left[(\dot{\theta}_{oe} - \dot{\theta}_{o}) \right.$$

$$\left. + (\ddot{\theta}_{oe} - \ddot{\theta}_{o}) t_n - \dot{\theta}_{N_n} \right]^{1} \cdot t_n \qquad \text{(A-6)}$$

or

$$\sum_{n=1}^{nx} \left[(\dot{\theta}_{oe} - \dot{\theta}_{o}) + (\ddot{\theta}_{oe} - \ddot{\theta}_{o}) t_n \right]$$

$$= \sum_{n=1}^{nx} \dot{\theta}_{N_n} \qquad \text{(A-7)}$$

$$\sum_{n=1}^{nx} \left[(\dot{\theta}_{oe} - \dot{\theta}_{o}) + (\ddot{\theta}_{oe} - \ddot{\theta}_{o}) t_n \right] t_n$$

$$= \sum_{n=1}^{nx} \dot{\theta}_{N_n} \cdot t_n \qquad \text{(A-8)}$$

Define the following:

$$\dot{\theta}_o D \equiv \dot{\theta}_{oe} - \dot{\theta}_o \qquad \text{(A-9)}$$

and

$$\ddot{\theta}_o D \equiv \ddot{\theta}_{oe} - \ddot{\theta}_o \qquad \text{(A-10)}$$

Then from (A-5), (A-6), (A-7) and (A-8), one has

$$\dot{\theta}_o D \cdot \left(\sum_{n=1}^{nx} 1 \right) + \ddot{\theta}_o D \sum_{n=1}^{nx} t_n = \sum_{n=1}^{nx} \dot{\theta} N_n \qquad \text{(A-11)}$$

$$\dot{\theta}_o D \cdot \left(\sum_{n=1}^{nx} t_n \right) + \ddot{\theta}_o D \cdot \sum_{n=1}^{nx} t_n{}^2 = \sum_{n=1}^{nx} \dot{\theta} N_n \cdot t_n \qquad \text{(A-12)}$$

Let

$$\text{Det} = \begin{vmatrix} nx & \sum_{n=1}^{nx} t_n \\ \sum_{n=1}^{nx} t_n & \sum_{n=1}^{nx} t_n{}^2 \end{vmatrix} = nx \cdot \left(\sum_{n=1}^{nx} t_n{}^2 \right) - \left(\sum_{n=1}^{nx} t_n \right)^2 \qquad \text{(A-13)}$$

Then

$$\dot{\theta}_o D = \frac{\begin{vmatrix} \sum_{n=1}^{nx} \dot{\theta} N_n & \sum_{n=1}^{nx} t_n \\ \sum_{n=1}^{nx} \dot{\theta} N_n \cdot t_n & \sum_{n=1}^{nx} t_n{}^2 \end{vmatrix}}{\text{Det}} \qquad \text{(A-14)}$$

and

$$\ddot{\theta}_o D = \frac{\begin{vmatrix} nx & \sum_{n=1}^{nx} \dot{\theta} N_n \\ \sum_{n=1}^{nx} t_n & \sum_{n=1}^{nx} \dot{\theta} N_n \cdot t_n \end{vmatrix}}{Det} \tag{A-15}$$

From (A-9), (A-10), (A-14) and (A-15), then one has

$$\dot{\theta}_{oe} = \dot{\theta}_o + \frac{1}{Det}\left[\sum_{n=1}^{nx} \dot{\theta} N_n \cdot \left(\sum_{n=1}^{nx} t_n^2\right) - t_n \cdot \sum_{n=1}^{nx} t_n\right] \tag{A-16}$$

$$\ddot{\theta}_{oe} = \ddot{\theta}_o + \frac{1}{Det}\left[\sum_{n=1}^{nx} \dot{\theta} N_n \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n\right)\right] \tag{A-17}$$

(A-16) and (A-17) are results for any specific replication.

Let TF be the time from first measurement to fire, then

$$\dot{\theta}_{eTF} = \dot{\theta}_{oe} + \ddot{\theta}_{oe} \cdot TF \tag{A-18}$$

Substitute (A-16) and (A-17) into (A-18), one has

$$\dot{\theta}_{eTF} = \dot{\theta}_o + \ddot{\theta}_o \cdot TF + \frac{1}{Det} \sum_{n=1}^{nx} \dot{\theta} N_n \left[\left(\sum_{n=1}^{nx} t_n^2\right) - t_n \cdot \sum_{n=1}^{nx} t_n \right.$$

$$\left. + \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n\right) \cdot TF\right] \tag{A-19}$$

Define $E\dot{\theta}$, the error ratio, as follows:

$$E\dot{\theta} = (\dot{\theta}P - \dot{\theta})/\dot{\theta} \tag{A-20}$$

Since

$$\dot{\theta}P = \dot{\theta}_{eTF} \tag{A-21}$$

and

$$\dot{\theta} = \dot{\theta}_o + \ddot{\theta}_o \cdot TF \tag{A-22}$$

Substitution of (A-21) and (A-22) into (A-20), yields

$$E\dot{\theta} = \frac{\dot{\theta}_{eTF} - (\dot{\theta}_o + \ddot{\theta}_o \cdot TF)}{\dot{\theta}} \tag{A-23}$$

Also equation A-19 can be written as

$$\dot{\theta}_{eTF} - (\dot{\theta}_o + \ddot{\theta}_o \cdot TF) = \frac{1}{Det} \sum_{n=1}^{nx} \dot{\theta}_{N_n}$$

$$\left[\left(\sum_{n=1}^{nx} t_n^2\right) - t_n \cdot \sum_{n=1}^{nx} t_n + \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n\right) TF\right] \tag{A-24}$$

Substitute (A-24) into (A-23), one obtains

$$E\dot{\theta} = \frac{\frac{1}{Det} \sum_{n=1}^{nx} \theta_{N_n} \left[\left(\sum_{n=1}^{nx} t_n^2\right) - t_n \sum_{n=1}^{nx} t_n + \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n\right) TF\right]}{\dot{\theta}} \tag{A-25}$$

If one lets

$$C_n = \left(\sum_{n=1}^{nx} t_n^2\right) - t_n \cdot \left(\sum_{n=1}^{nx} t_n\right) + \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n\right) TF \tag{A-26}$$

and

$$\text{Det} = nx \cdot \sum_{n=1}^{nx} t_n^{\,2} - \left(\sum_{n=1}^{nx} t_n \right)^2 \qquad \text{(A-13)}$$

Substituting (A-26) into (A-25), yields

$$E\dot{\theta} = \frac{1}{\text{Det}} \frac{\sum_{n=1}^{nx} C_n \quad \dot{\theta}N_n}{\dot{\theta}} \qquad \text{(A-27)}$$

Equations (A-27), (A-26), and (A-13) are similar to equations (2), (3), and (4) in the text.

APPENDIX B. DERIVATION OF ERROR IN LINE-OF-SIGHT RATE OF ACCELERATION

Similar to appendix A and by using equation (A-17) in appendix A

$$\ddot{\theta}_{oe} = \ddot{\theta}_o + \frac{1}{\mathrm{Det}} \left[\sum_{n=1}^{nx} \dot{\theta}_{Nn} \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n \right) \right] \qquad \text{(A-17)}$$

Define $\ddot{E\theta}$, the error ratio, as follows:

$$\ddot{E\theta} = (\ddot{\theta}P - \ddot{\theta})/\ddot{\theta} \qquad \text{(B-1)}$$

also

$$\ddot{\theta}P = \ddot{\theta}_{oe} \qquad \text{(B-2)}$$

and

$$\ddot{\theta} = \ddot{\theta}_o \qquad \text{(B-3)}$$

The line-of-sight acceleration is assumed to be constant with time. Substitute (B-2) and (B-3) into (B-1). Therefore,

$$\ddot{E\theta} = (\ddot{\theta}_{oe} - \ddot{\theta}_o)/\ddot{\theta} \qquad \text{(B-4)}$$

and equation (A- 17) can also be written as

$$\ddot{\theta}_{oe} - \ddot{\theta}_o = \frac{1}{\mathrm{Det}} \left[\sum_{n=1}^{nx} \dot{\theta}_{Nn} \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n \right) \right] \qquad \text{(B-5)}$$

Substitute (B- 5) into (B- 4), one obtains

$$\ddot{E\theta} = \frac{\frac{1}{\mathrm{Det}} \left[\sum_{n=1}^{nx} \dot{\theta}_{Nn} \left(t_n \cdot nx - \sum_{n=1}^{nx} t_n \right) \right]}{\ddot{\theta}} \qquad \text{(B-6)}$$

Let

$$CTDD_n = t_n \cdot nx - \sum_{n=1}^{nx} t_n \qquad \text{(B-7)}$$

Substitute (B-7) into (B- 6), one obtains

$$E\ddot{\theta} = \frac{1}{\ddot{\theta} \cdot Det} \sum_{n=1}^{nx} \dot{\theta}_{Nn} \cdot CTDD_n \qquad \text{(B-8)}$$

Note that equations (B-7) and (B-8) are similar to equations 28 and 29 in the main part of the report.

LIST OF SYMBOLS

α	Angle between VT and line-of-sight from launcher (stationary) through the target at time of fire or X axis.
α'	Angle between acceleration direction of target, at time of fire, and X axis.
$\dot{\theta}$	Line of sight rate at time of fire.
$\dot{\theta}_{en}$	n-th value of $\dot{\theta}$, estimated.
$\dot{\theta}_{eTF}$	$\dot{\theta}$ estimated at time of fire, TF.
$\dot{\theta}_{Mn}$	n-th value of $\dot{\theta}$, measured.
$\dot{\theta}_{Nn}$	$\dot{\theta}$ noise contribution for the n-th measurement.
$\dot{\theta}_{o}$	True initial value of $\dot{\theta}$ at time of first measurement.
$\dot{\theta}_{oe}$	Initial estimated value of $\dot{\theta}$ of target.
$\ddot{\theta}$	Line-of-sight acceleration rate (first derivative with time of $\dot{\theta}$).
$\ddot{\theta}_{en}$	n-th estimate of $\ddot{\theta}$.
$\ddot{\theta}_{o}$	True value of $\ddot{\theta}$ of target.
$\ddot{\theta}_{oe}$	Estimated value of $\ddot{\theta}$ of target.
A	Acceleration of target.
AP	Angle between velocity of projectile and X axis.
C_n and $CTDD_n$	Defined in equations 3 and 29.
Det	Value of determinent, defined in equation 4.
Dt	Time between each measurement.
$E\dot{\theta}$	Error ratio in $\dot{\theta}$ (equations 2 and 8).

$E\ddot{\theta}$	Error in θ double dot, divided by $\ddot{\theta}$ (equations 9 and 28).
I	Denotes component in direction of X axis.
J	Denotes component perpendicular to X axis.
K_1, K_2	Constants defined in equations 25 and 26.
MD	Miss Distance between projectile and launcher (in direction perpendicular to X axis).
NMAX or nx	Maximum number of measurements.
P	Perceived value of the term preceding P; perceived by the launcher either from measurement or the launcher's calculations.
SIGMA MD	Sigma miss distance.
t_n	Time of the nth measurement.
TF	Time from first measurement in prelaunch phase to fire.
TINT	Time to intercept.
VP	Velocity of projectile.
VT	Velocity of target.
VTJ	Velocity of target in the J (perpendicular) to X axis) direction.
X_o	Distance from launcher to target at time of fire.

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